Lista 2: Limites

Equivalências & $\longrightarrow$ 0 por algum motivo.

1) sen & ~ &	2) ln (1+&) ~&
3) $e^{\&}$ - 1 ~ &	4) tg & ~ &
5)arc sem & ~ &	6) arc tg & ~ &
7) 1- cos & ~ $\&^2/2$	8) $(1+\&)^m$ – 1 ~ m&
9) $\sqrt[q]{1-\&} - 1 \sim \&/q$	

Ex:

1) $\lim\limits_{x \to 0} \frac{sen\, x}{x} = 1$; 2) $\lim\limits_{x \to 0} \frac{ln(1+x)}{x} = 1$; 3) $\lim\limits_{x \to 0} \frac{e^x - 1}{x} = 1$

Exercícios:

1) $\lim\limits_{x \to 0} \frac{sen^3\, x}{x^3}$

2) $\lim\limits_{x \to 0} \frac{sen\, 5x}{ln(1+4x)}$

3) (ANPEC 99) $\lim\limits_{x \to 0} \frac{e^3 - 1}{sen\, x}$

4) $\lim\limits_{x \to 0} \frac{1 - cos\, 5x}{x^2}$

5) $\lim\limits_{x \to 0} \frac{ln(1+mx)}{x}$

6) $\lim\limits_{x \to 0} \frac{ln(x+2) - ln\, 2}{x}$

Prove que:

1º) $\lim\limits_{x\to 0} \frac{a^x - 1}{x} = \ln a$

2º) $\lim\limits_{x\to 0} \frac{\log_a(1+x)}{x} = \log a^e$ se $f = a^x \Rightarrow f' = a^x \ln a$

logo, $f = \log_a g \Rightarrow f' = \frac{g'}{g \ln a} \ln a$

3º) $\lim\limits_{x\to 0} \frac{(e^{\ln a})^x - 1}{x} = \lim\limits_{x\to 0} \frac{e^{x \ln a} - 1}{x} = \frac{x \ln a}{x} = \ln a$

4º) $\lim\limits_{x\to 0} \frac{\frac{\log e^{(1+x)}}{\log e^a}}{x} = \frac{1}{\log e^a} . \lim\limits_{x\to 0} \frac{\log e^{(1+x)}}{x} = \log_a e$

Perfume de "e"

$$\lim_{z\to 0} (1+z)^{1/z} = e$$

$$\lim_{v\to\infty} \left(1+\frac{1}{v}\right)^v = e$$

Exercícios Resolvidos

1) $\lim\limits_{x\to\infty} \left(\frac{x+6}{x}\right)^x = \lim\limits_{x\to\infty} \left[\left(1+\frac{6}{x}\right)^{x/6} \right]^6 = e^6$

2) $\lim\limits_{x\to\infty} \left(1+\frac{3}{x}\right)^x = \lim\limits_{x\to\infty} \left[\left(1+\frac{3}{x}\right)^{x/3} \right]^3 = e^3$

3) $\lim_{x\to\infty}(1+\frac{1}{x})^{7x} = \lim_{x\to\infty}\left[\left(1+\frac{1}{x}\right)^{x}\right]^{7} = e^{7}$

4) $\lim_{x\to\infty}(1+\frac{1}{x})^{x+a} = \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x} . \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{a} \Rightarrow e \, . \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{a} \Rightarrow e.1 = e$

5) $\lim_{x\to\infty}(1+\frac{1}{x-3})^{x} \Rightarrow \begin{cases} x-3=t \\ x=t+3 \\ x\to\infty \Rightarrow t\to\infty \end{cases} \Rightarrow \lim_{t\to\infty}(1+\frac{1}{t})^{t+3} = e$

ANPEC 2001) $\lim_{x\to\infty}(1+\frac{3}{x})^{x/5} = e^{5/3}$?

Qual é o método? Fazemos aparecer o "e" e depois "arrumamos" para não modificar a questão, assim:

$\lim_{x\to\infty}(1+\frac{3}{x})^{x/5} \Rightarrow \lim_{x\to\infty}\left[\left(1+\frac{3}{x}\right)^{x/3}\right]^{3/5} = e^{3/5}$ *Falso*

ANPEC 2002) $\lim_{x\to\infty}(1+\frac{3}{x})^{2x+5} = e^{5}$?

Da mesma forma:

$\lim_{x\to\infty}(1+\frac{3}{x})^{2x+5} \Rightarrow \lim_{x\to\infty}\left[\left(1+\frac{3}{x}\right)^{x/3}\right]^{6} . \lim_{x\to\infty}(1+\frac{3}{x})^{5} = e^{6}.1 = e^{6}$